Supporting information for Design of a fiber-cavity ion-trap for a high efficiency and high rate quantum network node

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A common ion-cavity system can be described by the parameters (g, κ, γ), where g is the coherent interaction rate between the particle and cavity, κ is the HWHM (half-width half-maximum) of the cavity transmission line (intracavity field decay at a rate 2κ), and γ is the FWHM (full-width half-maximum) of the spontaneous line (spontaneous decay at a rate γ).

Note that

$$\kappa = \frac{2\pi\delta\nu}{2} \approx \frac{c[2-(1-\mathcal{L}_\mathrm{c})(\mathcal{R}_l+\mathcal{R}_r)]}{4n_c L}, \tag{1}$$

where $\delta\nu$ is the FWHM frequency of the cavity transmission line, $\mathcal{L}_{\mathrm{c}}$ is the single-pass effective cavity loss, L is the cavity length, n_{c} is the refractive index of the cavity, and $\mathcal{R}_l$ and $\mathcal{R}_r$ are the reflectivity of the left and right cavity mirrors, respectively. In this equation, $[2-(1-\mathcal{L}_{\mathrm{c}})(\mathcal{R}_{\mathrm{l}}+\mathcal{R}_{\mathrm{r}})]$ represents an approximate round-trip loss, and this approximation only holds when the cavity loss $\mathcal{L}_{\mathrm{c}}$ is very small and the reflectivity of the mirror $\mathcal{R}_{\mathrm{l}}$ or $\mathcal{R}_{\mathrm{r}}$ is very large.

It is worth noting that the intensity of a single reflection is affected by the clipping loss because the size of the cavity surface of the fiber cavity is finite. The clipping loss is defined as

$$\mathcal{L}_{\mathrm{cl}} = \exp\left(-\frac{D^2}{2w_{\mathrm{m}}^2}\right), \tag{2}$$

where w_{m} is the radius of the cavity mode at the position of the fiber surface, and D is the diameter of the effective reflective surface.

The coupling coefficient g is evaluated by

$$g = \frac{\mu_{\mathrm{d}} E}{\hbar}, \tag{3}$$

where μ_{d} is the electric dipole moment of the particle and E is the electric field strength of a single photon at the position of the particle. In general, the single-photon electric field intensity is

$$E = \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}\psi(\vec{r}_a), \tag{4}$$

where $V = \pi w_0^2 L/4$ is the modal volume, w_0 is the waist radius, $\vec{r}_a$ is the position of the ion and $\psi(\vec{r})$ is the normalized cavity mode distribution where $\max_{\vec{r}} |\psi(\vec{r})| = 1$. Using the Gaussian mode to approximate the cavity mode, the size of the beam waist satisfies $w_0^2 = z_0\lambda/\pi$, where z_0 is the Rayleigh length and λ is the wavelength.

Define the single-atom cooperativity parameter $C_1 = g^2/\kappa\gamma$. Then, the quantum efficiency of single-photon sources η_q in the bad cavity regime ($\kappa \gg g^2/\kappa \gg \gamma$) is given by

$$\eta_{\mathrm{q}} = \frac{2C_1}{2C_1+1}. \tag{5}$$

In the strong regime ($g \gg \kappa, \gamma$), the quantum efficiency of single-photon sources η_{q} is given by

$$\eta_{\mathrm{q}} = \frac{2C_1}{2C_1+1}\frac{2\kappa}{2\kappa+\gamma}. \tag{6}$$

By changing the structure of the cavity, we can change the values of g and κ and thus achieve Purcell enhancement.

κ in the above equation contains many additional transmission channels and loss channels, while the available single photons are only transmitted in a specific transmission channel. Therefore, it is necessary to consider the percentage of transmission in the total channels, that is,

$$\eta_{\mathrm{t}} = \frac{\mathcal{T}_o}{2-(1-\mathcal{L}_{\mathrm{c}})(\mathcal{R}_{\mathrm{l}}+\mathcal{R}_{\mathrm{r}})}, \tag{7}$$

where $\mathcal{T}_o$ is the transmissivity of the output channel.

In addition to η_q, the matching between cavity modes and fiber modes also limits the overall coupling efficiency. This matching efficiency η_{fc} is given by

$$\eta_{\mathrm{fc}} = \frac{4}{\left(\frac{w_{\mathrm{f,o}}}{w_{\mathrm{m,o}}}+\frac{w_{\mathrm{m,o}}}{w_{\mathrm{f,o}}}\right)^2+\left(\frac{\pi n_{\mathrm{f,o}} w_{\mathrm{f,o}} w_{\mathrm{m,o}}}{\lambda R_{\mathrm{o}}}\right)^2}, \tag{8}$$

where $n_{\mathrm{f,o}}$ is the refractive index of the output fiber, $w_{\mathrm{f,o}}$ is the waist radius of the corresponding fiber mode, $w_{\mathrm{m,o}}$ is the radius of the cavity mode at the position of the mirror of the output fiber and R_{o} is the radius of curvature of this mirror.

In this system, the tunable parameters are the cavity length L, coating parameters $(\mathcal{R}, \mathcal{T}, \mathcal{L})$, structure of the cavity, and type of fiber.